

## A Simple Circuit Model for Resonant Mode Coupling in Packaged MMICs

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### Abstract

Neglecting the effect of enclosing an MMIC circuit in a resonant conducting package can have undesirable consequences such as power loss, poor isolation, and circuit instabilities. In principle these effects can be predicted by currently available full-wave CAD programs. In practice, however, such programs are difficult to implement for a complex circuit and are very CPU intensive when realistic box/circuit sizes are used. A simpler approach would be useful. In this paper we will develop a simple circuit model which will predict coupling effects between various circuit components due to a package resonance. It is easily implemented on commercially available CAD packages and requires several orders of magnitude less CPU time than a full-wave technique.

### Introduction

Recently, the effect of enclosing an MMIC circuit in a conducting package has received considerable attention. MMICs are generally enclosed in a package to reduce outside electromagnetic interference and to isolate one circuit from another. However, enclosing a circuit can have some undesirable effects, namely, parasitic coupling to resonant modes of the enclosure. Resonant mode coupling can result in catastrophic coupling between different elements of a circuit which might otherwise have been isolated. Williams [1] investigated the use of lossy materials to reduce resonant mode coupling. Burke and Jackson [2] have shown that an additional reduction in resonant mode coupling can be achieved by repositioning a circuit in the enclosure.

A variety of full-wave techniques have been developed to analyze MMIC circuits in an enclosure [2, 3, 4, 5]. A full-wave analysis is numerically rigorous and therefore it includes parasitic coupling to resonant modes, but the cost of this accuracy is increased CPU time and complexity. Consequently a simpler approach is required.

Toward that end, Jansen and Wiemer [6] have developed a simple circuit theory model to describe coupling of circuit junctions to a resonant mode. Their model appears to be based on the assumption that the resonant mode coupling occurs only at a discontinuity.

In contrast, Lewin [7] has shown that the interaction of a circuit with space wave radiation and surface waves can occur at more than a guide wavelength from a discontinuity. Although this phenomena is for a circuit with no cover plate and side walls, Burke and Jackson [2] have shown a similar effect occurs for a circuit in an enclosure. These results suggest that resonant mode coupling can be modeled in a way which does not strictly tie it to discontinuities.

In this paper we will develop a simple circuit model to accurately describe resonant mode coupling between circuit elements within an MMIC enclosure. This model is appropriate for use on commercially available CAD packages.

### Theory

To implement the resonant mode circuit model, a circuit is divided into several segments. In series with each segment, the primary of a coupling transformer is inserted. The secondaries of all the transformers for a particular mode are all connected in shunt with a single shunt  $RLC$  tank circuit. In an enclosure with multiple modes, a separate set of transformers and a tank circuit are required for each mode. The turns ratio of each transformer is a function of the corresponding segment's location within the package, its orientation, and the mode being modeled.

To illustrate the use of the resonant mode coupling circuit consider a length of a microstrip transmission line. The transmission line is divided into  $N$  segments of length  $\Delta L$  and a coupling transformer is inserted between each two adjoining segments. The resulting circuit is shown in Figure 1. If we assume there is only one resonant mode, then the one tank circuit shown is the only one needed. The characteristics of the tank circuit are independent of the number of transformers.

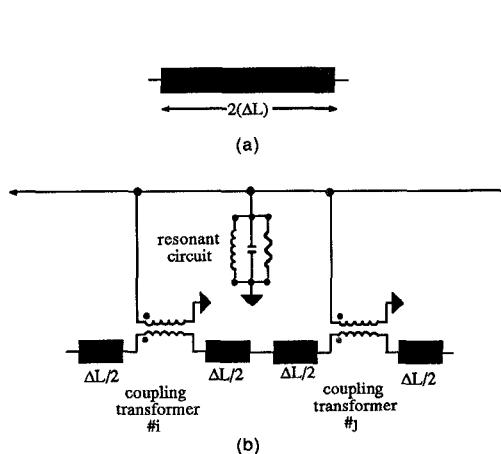


Fig. 1 Schematic of (a) a microstrip transmission line and (b) the proposed circuit resonant mode coupling circuit model.

### Development of The Circuit Model

For the circuit model in figure 1, the mutual impedance,  $Z_{ij}^{\text{model}}$ , between the  $i$ th and  $j$ th current elements of the transmission line can be written as:

$$Z_{ij}^{\text{model}} = Z_{ij}^{\text{cir}} + \frac{n_i n_j}{\frac{1}{R} + j \frac{C}{\omega} (\omega^2 - \omega_0^2)} = Z_{ij}^{\text{cir}} + \frac{n_i n_j}{Y_R} \quad (1)$$

where  $Z_{ij}^{\text{cir}}$  is the mutual impedance between the two circuit elements if there were no resonant mode coupling present,  $n_i$  is the turns ratio for the  $i$ th coupling transformer and  $Y_R$  is the admittance of the  $RLC$  tank circuit. The turns ratio,  $n_i$ , is defined as the number of turns in the primary to the number of turns in the secondary. The resonant frequency,  $\omega_0$ , is given by:

$$\omega_0^2 = 1/(LC) \quad (2)$$

Expressions for the turns ratio of the coupling transformers and the  $R$ ,  $L$ , and  $C$  components of the tank circuit are determined via a comparison between the rigorous full wave mutual impedance,  $Z_{ij}^{FW}$ , and the circuit model impedance,  $Z_{ij}^{\text{model}}$ .

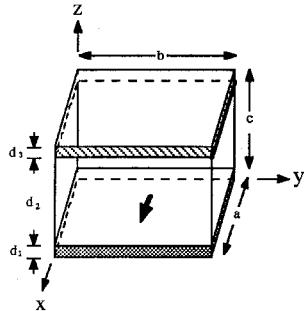


Fig. 2 Geometry used in the derivation of the Green's function.

For the package illustrated in Figure 2 with three dielectric layers (layer 2 is free space), the full-wave mutual impedance between the  $i$ th and  $j$ th infinitesimal  $x$  directed current elements on the surface of the first layer is given [2] by:

$$Z_{ij}^{FW} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{4\epsilon_n \epsilon_m}{ab} \left[ \frac{k_{xn}^2}{k_p^2} Q_{TM} + \frac{k_{ym}^2}{k_p^2} Q_{TE} \right] \cdot [\Delta_L T_x(x_i, y_i)] [\Delta_L T_x(x_j, y_j)] \quad (3)$$

$$T_x(x, y) = \cos(k_{xn}x) \sin(k_{ym}y) \quad (4)$$

$$Q_{TV} = \frac{1}{Y_{LV}^{(1)} + Y_{RV}^{(2)}} = \frac{1}{Y_V} \quad (5)$$

$$Y_{LV}^{(1)} = -jY_{TV}^{(1)} \cot(k_{z1}d_1) \quad Y_{RV}^{(3)} = -jY_{TV}^{(3)} \cot(k_{z3}d_3)$$

$$Y_{RV}^{(2)} = Y_{TV}^{(2)} \frac{Y_{RV}^{(3)} + jY_{TV}^{(2)} \tan(k_{z2}d_2)}{Y_{TV}^{(2)} + jY_{RV}^{(3)} \tan(k_{z2}d_2)}$$

$$Y_{TM}^{(i)} = \frac{\epsilon_{ri} k_0}{k_{zi} \eta_0} \quad Y_{TE}^{(i)} = \frac{k_{zi}}{\mu_{ri} k_0 \eta_0}$$

$$k_{zi}^2 = \epsilon_{ri} \mu_{ri} k_0^2 - k_p^2 \quad \text{Im}(k_{zi}) < 0$$

$$k_p^2 = k_{xn}^2 + k_{ym}^2 \quad k_{xn} = \frac{n\pi}{a} \quad k_{ym} = \frac{m\pi}{b}$$

$$\epsilon_k = \begin{cases} 0.5 & k = 0 \\ 1.0 & k \neq 0 \end{cases} \quad V=E \text{ or } M, i=1, 2, \text{ or } 3$$

Near a resonance of the enclosure  $Z_{ij}^{FW}$  can be written as:

$$Z_{ij}^{FW} = Z_{ij}^{NR} + Z_{ij}^R \quad (6)$$

where  $Z_{ij}^{NR}$  is the non-resonant part of  $Z_{ij}^{FW}$ , and the resonant part of  $Z_{ij}^{FW}$ ,  $Z_{ij}^R$ , is given by:

$$Z_{ij}^R = \frac{4\epsilon_n \epsilon_m}{ab} \frac{k_{xn}^2}{k_p^2} Q_{TM} [\Delta_L T_x(x_i, y_i)] [\Delta_L T_x(x_j, y_j)] \quad (7)$$

where it is assumed that only a particular  $TM_{nm}$  mode is resonant and that near resonance  $Z_{ij}^R$  dominates the other terms of the summation in (3). In a typical enclosure housing an MMIC chip, the cover height is low enough that only  $TM$  modes are resonant over the frequency of operation. However, this modeling procedure could easily be applied to a  $TE$  mode resonance. Near a resonance, in an enclosure with small to moderate loss,  $Q_{TM}$  can be approximated by the following:

$$Q_{TM} \approx \frac{-j2\omega}{\left[ \text{Im} \left( \frac{\partial Y_M}{\partial \omega} \right) \right]_{\omega=\omega_0} \left( \omega^2 - \omega_0^2 - j2\omega\omega_r'' \right)} \quad (8)$$

where the enclosure resonant frequency is

$$\omega_r = \omega_r' + j\omega_r'' = \omega_0 + j\omega''$$

It should be emphasized that when the enclosure contains loss, the resonant frequency becomes complex. The solution for  $\omega_r$  will be discussed shortly.

In the limit as  $\Delta_L$  becomes small, equations (1) and (6) are approximately equal. By comparing like terms in each equation, one may write,

$$Z_{ij}^{NR} \approx Z_{ij}^{\text{cir}} \quad (9)$$

$$Z_{ij}^R \approx \frac{n_i n_j}{\frac{1}{R} + j \frac{C}{\omega} (\omega^2 - \omega_0^2)} = \frac{n_i n_j}{Y_R} \quad (10)$$

By inserting (7) in (10),  $n_i$  and  $Y_R$  can be identified as:

$$n_i = \sqrt{\frac{4\epsilon_n \epsilon_m}{ab} \frac{k_{xn}}{k_p} [\Delta_L \cos(k_{xn}x_i) \sin(k_{ym}y_i)]} \quad (11)$$

$$\frac{1}{\frac{1}{R} + j \frac{C}{\omega} (\omega^2 - \omega_0^2)} = Q_{TM} \quad (12)$$

Substituting equation (8) into the right side of equation (12) and solving for  $R$  and  $C$  yields,

$$C = \frac{1}{2} \text{Im} \left( \frac{\partial Y_M}{\partial \omega} \right) \Big|_{\omega=\omega_0} \quad (13)$$

$$R = \frac{1}{2\omega_r'' C} \quad (14)$$

The inductance,  $L$ , of the tank circuit is given by equation (2). An alternative expression for  $R$  may be derived by substituting equation (5) into equation (12) and solving for  $R$  when  $\omega=\omega_0$  thus  $R$  is given by:

$$R = Q_{TM}(\omega_0) = \frac{1}{Y_M(\omega_0)} \quad (15)$$

The  $Q$  of the enclosure is given by:

$$Q = \omega_0 CR = \frac{\omega_0}{2\omega''_r} \quad (16)$$

Thus, simple expressions have been developed for  $n_i$ ,  $R$ ,  $L$ , and  $C$  in terms of the circuit location and the electrical characteristics of the enclosure. In the previous derivation we have assumed that the circuit elements are all  $x$  oriented. For coupling between a circuit element which is  $x$  oriented and one which is  $y$  oriented, the only difference is that the expression for the  $y$  current turns ratio is modified. The new expression is easily derived in a manner similar to what was shown above and results in an interchange of  $x_i$  with  $y_i$  and  $k_{xn}$  with  $k_{ym}$  in equation (11).

### Numerical Evaluation of Circuit Components

First, the resonant frequencies of the enclosure must be determined. The poles of  $Q_{TM}$  (eq. 5) correspond to the resonant frequencies of the cavity and can be located by finding the zeros of  $Y_M$ . For a lossless cavity,  $Y_M$  is purely imaginary and  $\omega_r$  is purely real. The zeros of  $Y_M$  can be located by searching for the frequency at which  $\text{Im}(Y_M)=0$ . If loss is present in the cavity,  $Y_M$  is complex and the resonant frequency of enclosure,  $\omega_r$ , is also complex. The zeros of  $Y_M$  are located by searching for a complex  $\omega$  where the real and imaginary parts of  $Y_M$  simultaneously equal zero. Searching for the real  $\omega$  where  $\text{Im}(Y_M)=0$  may result in a solution which is not an actual zero of  $Y_M$  (i.e.  $\text{Re}(Y_M)\neq 0$ ). Therefore, in general, the zeros should not be located by searching for a frequency where only one part of  $Y_M$  equals zero. But, for cavities with small to moderate loss, the real part of the resonant frequency can be determined with good accuracy if the approximate solution satisfies both of the following conditions:

$$\text{Im}(Y_M) \Big|_{\omega=\omega_0} = 0 \quad (17)$$

$$\text{Im}\left(\frac{\partial Y_M}{\partial \omega}\right) \Big|_{\omega=\omega_0} > 0 \quad (18)$$

After finding the real frequency,  $\omega_0$ , under these conditions, the components of the  $RLC$  tank circuit can be determined by equations (2), (13) and (15).

For example, consider a cavity of the following dimensions:  $a=15$  mm,  $b=24$  mm and  $c=12.7$  mm. The substrate thickness is  $d_1=1.27$  mm and the relative permittivity is  $\epsilon_{r1}=10.5(1-j0.0023)$ . In the band 9-12 GHz, there is one real frequency for which  $\text{Im}(Y_M)=0$ . This frequency has been determined to be  $f_0=10.8129$  GHz. Substituting  $f_0$  into equations (2), (13) and (15) yields  $R=0.1$  M $\Omega$ ,  $C=0.615$  pF and  $L=0.352$  nH. For comparison, solving for the exact complex resonant frequency of the enclosure yields  $f_r=(10.8129-j0.00129)$  GHz.

A dielectric layer of thickness  $d_3=0.381$  mm and relative permittivity of  $\epsilon_{r3}=11.9(1-j25)$  was attached to the cover of the enclosure. In the band 9-12 GHz, there is one real frequency for which  $\text{Im}(Y_M)=0$ . This frequency has been determined to be  $f_0=10.7417$  GHz. Substituting  $f_0$  into equations (2), (13) and (15) yields  $R=2.786$  k $\Omega$ ,  $C=0.655$  pF and  $L=0.335$  nH. The  $Q$  of this enclosure was determined to be equal to 123. Solving for the complex resonant frequency of the enclosure yields  $f_r=(10.7381-j0.0439)$  GHz. Using equation (14), the resistor in the tank circuit is  $R=2.718$  k $\Omega$ . Searching for the resonance of the enclosure using both methods yields similar results. Therefore, the first method is valid even for enclosures with  $Q$ 's as low as 100.

Next, a microwave absorbing layer was attached to the cover of the enclosure. The thickness of this layer is  $d_3=1.27$  mm, the relative permittivity is  $\epsilon_{r3}=21(1-j0.02)$  and the relative permeability is  $\mu_{r3}=1.1(1-j1.4)$ . In the band 9-12 GHz, there is one real frequency for which  $\text{Im}(Y_M)=0$ . This frequency has been determined to be  $f_0=11.1325$  GHz. Substituting  $f_0$  into equations (2), (13) and (15) yields  $R=623$   $\Omega$ ,  $C=0.416$  pF and  $L=0.491$  nH. Solving for the complex resonant frequency of the enclosure yields  $f_r=(11.0508-j0.2497)$  GHz.

After determining the tank circuit components for each resonant mode of the enclosure, the circuit to be analyzed is divided up into segments of length  $\Delta L$ . In series with each segment, the primary of a coupling transformer is inserted. The turns ratio,  $n_i$ , for a coupling transformer located at  $(x_i, y_i)$  is given by equation (11). The secondaries of all the transformers for a particular mode are connected to a shunt  $RLC$  tank circuit. It has been determined that a sufficient number of segments be used such that the length of each segment,  $\Delta L$ , should be less than a quarter guide wavelength.

## Results

To verify the accuracy of the proposed circuit model, two circuits were analyzed. The first circuit analyzed was a transmission line with a single shunt open circuit stub attached midway between the input and the output connectors. The second circuit analyzed consisted of a large gap in a transmission line. The scattering parameters for each circuit were determined using the proposed circuit model and compared to the results using the full-wave analysis outlined in [2].

### Shunt Stub

A transmission line with a single shunt open circuit stub attached was analyzed using the proposed circuit model. The stub is centered at  $x=7.5$  mm ( $a/2$ ) and has a length of 1.9 mm. The transmission line is located at  $y_c=12$  mm ( $b/2$ ). The width of the transmission line and the stub is 1.4 mm.

The circuit is enclosed in a cavity of the following dimensions:  $a=15$  mm,  $b=24$  mm and  $c=12.7$  mm. The substrate thickness is  $d_1=1.27$  mm and the relative permittivity is  $\epsilon_{r1}=10.5(1-j0.0023)$ . As was calculated in the previous section, an enclosure of this size has only one resonant mode, the  $TM_{110}$  (10.8129 GHz), in the band 9-12 GHz. The lumped elements  $R$ ,  $C$  and  $L$  were found to be 0.1 M $\Omega$ , 0.615 pF and 0.352 nH.

The transmission line on either side of the stub was divided into 4 segments and the stub was divided into 2 segments. The turns ratio for the 2 transformers used in the stub were determined using equation (11) by interchanging  $x_i$  with  $y_i$  and  $k_{xn}$  with  $k_{ym}$ . The Tee-junction itself is modeled using the Tee element in a typical CAD program.

Figure 3 compares the predicted transmission response of the stub in the high  $Q$  package using the circuit model to the full-wave analysis [2]. Agreement between the proposed circuit model and the full-wave analysis is good. Note that the resonant mode of the enclosure has a drastic effect on the circuit's performance and that the circuit model does in fact show this effect.

To reduce the coupling of power to the resonant mode, a microwave absorbing layer was attached to the cover of the enclosure [1, 2]. The thickness of this layer is  $d_3=1.27$  mm, the relative permittivity is  $\epsilon_{r3}=21(1-j0.02)$  and the relative permeability is  $\mu_{r3}=1.1(1-j1.4)$ . This cavity was also analyzed in the previous section and was found to have one resonant mode, the  $TM_{110}$  (11.13 GHz). The lumped elements  $R$ ,  $C$  and  $L$  were found to be 623  $\Omega$ , 0.416 pF and 0.491 nH.

Figure 4 compares the predicted transmission response of the stub in the low  $Q$  enclosure using the proposed circuit model to the full-wave analysis. Agreement between the circuit model and the full-wave analysis is good up to 11.5 GHz, but deteriorates above 11.5 GHz.

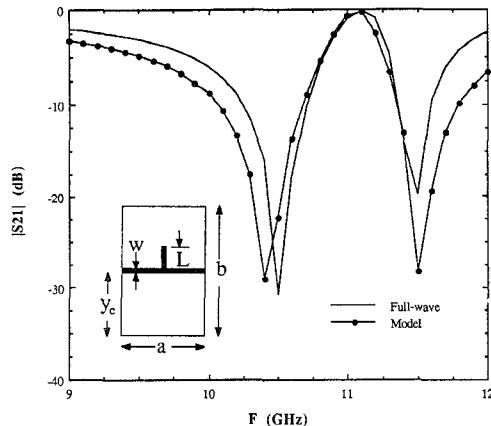


Fig. 3 Comparison of the computed  $|S_{21}|$  for the stub in the high  $Q$  package using the circuit model to the full-wave analysis.

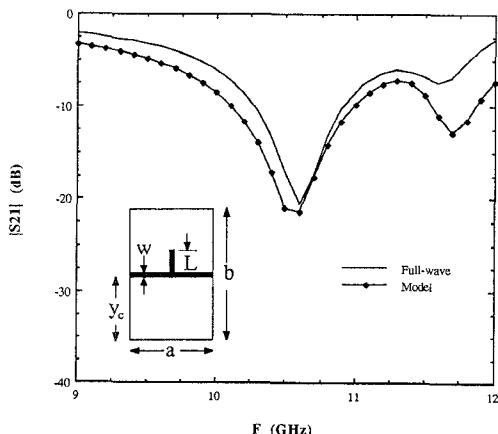


Fig. 4 Comparison of the computed  $|S_{21}|$  for the stub in the low  $Q$  package using the circuit model to the full-wave analysis.

### Large Gap

Consider a cavity of the following dimensions:  $a=30$  mm,  $b=48$  mm and  $c=10.0$  mm. The substrate thickness is  $d_1=1.27$  mm and the relative permittivity is  $\epsilon_r=10.5(1-0.0023)$ . In the band 4-9 GHz, the  $TM_{110}$  (5.505 GHz) and the  $TM_{120}$  (7.419 GHz) are resonant in such an enclosure. The lumped elements  $R, C$  and  $L$  were found to be  $0.112 \text{ M}\Omega$ ,  $0.425 \text{ pF}$  and  $0.197 \text{ nH}$  for the  $TM_{110}$  mode and  $0.114 \text{ M}\Omega$ ,  $0.196 \text{ pF}$  and  $0.234 \text{ nH}$  for the  $TM_{120}$  mode.

A transmission line of width 1.2 mm, located at  $y_c=24$  mm ( $b/2$ ), with a large gap,  $g=15$  mm, was analyzed using the proposed circuit model. Figure 5 shows the transmission response predicted by the circuit model and by the full-wave analysis. Agreement between the circuit model and the full-wave analysis is good.

The circuit model used for analysis of the two circuits discussed above takes several orders of magnitude less computation time than the full-wave analysis.

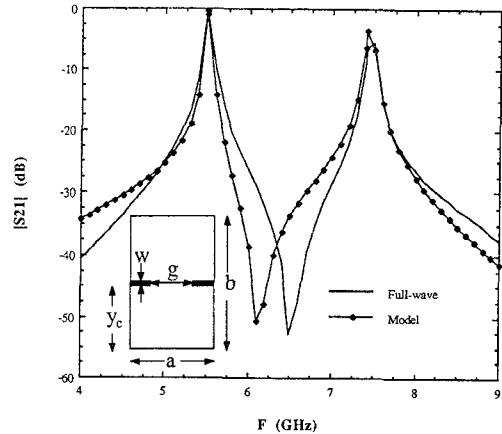


Fig. 5 Comparison of the computed  $|S_{21}|$  for a large gap in a transmission line using the circuit model to the full-wave analysis.

### Conclusion

A circuit model has been developed that models resonant mode coupling. It has good accuracy for enclosures with a  $Q$  of over 100 and is useful for  $Q$ 's as low as 20. For lower  $Q$ 's, the full wave spectral summation, which serves as the starting point for this model, no longer has a single spectral term which produces a dominant resonance.

The circuit model can be used on commercially available software packages. Simple analytical expressions for the entire circuit model are easily evaluated. The circuit model for an MMIC circuit in a large enclosure (one with more than a few resonances) may require a large number of transformers. Consequently, this circuit model is suited for MMIC circuits in moderately sized enclosures.

### Acknowledgement

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